Robust analysis towards robust optimization in engineering design

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Abstract — This paper deals with robust design analysis in preliminary design. The proposed approach considers design parameters variability. Towards a global robust and deterministic optimization we propose a method allowing to incorporate the uncertainties (Mean and Standard Deviation) as usual design parameters. We compare this method with classical Monte-Carlo simulations.

I. Introduction

This paper attempts to consider robust design problem. In real engineering, product design problem may be subject to various uncertainties appearing everywhere and cannot be avoided. Uncertainties mainly influence product performances and can lead to wrong products. Uncertainties may concern every design parameter, such as new environmental conditions, geometrical parameters, material properties and so on.

The main target of robust design is to control product performances taking parameter's variability into account. This present paper deals only with design parameter variations including material properties. We present in this paper how to fill in those objectives in a way that fit with optimization. Furthermore, in this work we consider that robust design will be attained in preliminary design stage dealing with analytical models. They come from FEM/RSM [1] or from approximation of physical laws.

The total time for finding a solution using optimization can be approximated by the product of the number of model evaluations and the time required for one evaluation. To decrease this total time we focus on the evaluation time of the model which integrates uncertainties.

II. Robust Analysis Methods

Robust design offers methods to make product solutions insensitive to variability sources. Given the variations of input design parameters, robust analysis methods are introduced to compute the variability of the product performances (output parameters). Several approaches can be used; they involve fuzzy set, interval variables, or random variables. We assume here that design parameters including material properties can be represented either by distributions or by nominal values.

Methods for robust analysis can be classified into two categories: simulation methods that give whole output parameter distributions and moments evaluation methods that estimate moments of the probability distribution. All moments expressions completely characterize the expected distribution.

A. Simulation methods

The most common simulation method is Monte Carlo simulations. This method can be easily implemented but it requires 10^4 to 10^6 samples to obtain accurate enough results [2].Time needed for complex model is too high. To reduce time for each model evaluation, Monte Carlo simulation can be applied to an approximate model such as the Taylor expansions or the Chaos polynomial [3]. It is also possible to reduce the number of evaluations using Latin Hypercube or Importance Sampling techniques.

B. Moments estimation

The Univariate dimension-reduction method [4] uses a moment-based quadrature rule for performing numerical integration to estimate moments. But analytic links between inputs and outputs parameters will be lost, as for simulation methods. Those links are necessary when dealing with deterministic optimization process. Furthermore, the Propagation of moments method [5] introduced in the next section gives analytic moments estimation.

C. The proposed approach

This last method is best suited to the optimization since it gives analytical formulas of the moments based on Taylor series expansion. Thus it keeps analytic links between inputs and outputs parameters. We have limited this method to the evaluation of the two first moments expressions μ_Y and ${\sigma_Y}^2$. Mathematical expressions for the Taylor first order (1) and for the Taylor second order (2) are:

$$\begin{cases} \mu_Y \approx h(\mu_X) \\ {\sigma_Y}^2 \approx \displaystyle\sum_{i=1}^{N} \left(\frac{\partial h}{\partial X_i}(\mu_X) \right)^2 {\sigma_i}^2 \end{cases} \quad (1)$$

$$\begin{cases} \mu_Y \approx h(\mu_X) + \dfrac{1}{2} \displaystyle\sum_{i=1}^{N} \frac{\partial^2 h}{\partial {X_i}^2}(\mu_X) {\sigma_i}^2 \\ {\sigma_Y}^2 \approx \displaystyle\sum_{i=1}^{N} \left(\frac{\partial h}{\partial X_i}(\mu_X) \right)^2 {\sigma_i}^2 + \frac{1}{2} \sum_{i=1}^{N} \left(\frac{\partial^2 h}{\partial X_i^2}(\mu_X) \right)^2 {\sigma_i}^4 \\ \qquad + \displaystyle\sum_{i<j}^{N} \left(\frac{\partial^2 h}{\partial X_i \partial X_j}(\mu_X) \right)^2 {\sigma_i}^2 {\sigma_j}^2 \end{cases} \quad (2)$$

Where h is the model, X_i are input parameters, Y are the output parameters, $\mu_X = (\mu_1, \dots, \mu_N)$ is the set of input parameter's Means (first moments) and ${\sigma_i}^2$ is the parameter variances (second moments) of X_i.

	Reference: Monte-Carlo 10^6		Propagation of moments method		Monte-Carlo 10^3		Monte-Carlo 10^4		Monte-Carlo 10^5	
$\sigma_i(\%)$	μ_{V_u} (10^{-4})	$\sigma_{V_u}(10^{-5})$	μ error (%)	σ error (%)	μ error (%)	σ error (%)	μ error (%)	σ error (%)	μ error (%)	σ error (%)
0,5	6,75	0,658	0	0,015	0,044	2,790	0	1,031	0,015	0,258
0,83	6,75	1,09	0	0	0,044	1,654	0,015	0,276	0,015	0,276
1,67	6,76	2,18	0	0	0,089	0,781	0,015	0,046	0	0,184
3,33	6,76	4,34	0,015	0,023	0,237	1,607	0,089	0,597	0,015	0,413
5,55	6,77	7,30	0	0,083	0,089	1,830	0,030	0,206	0,015	0,413

TABLE I

RELATIVE ERROR BETWEEN MONTE-CARLO (10^6), THE PROPAGATION OF MOMENTS METHOD AND OTHER MONTE-CARLO

The global model (Fig. 1) is made of the "initial" one to which we add the "model of moments" computed by (1) or (**2**). This new model is an approximate reformulated one.

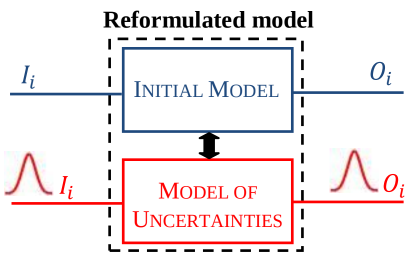

Fig. 1. Optimization strategy that incorporate uncertainties

Besides, the "reformulated model" is dedicated to optimization and so it must be accurate enough for Means, Standard Deviations and their derivatives.

In the next part we compare on accuracy's criterion the Propagation of moments method for the Taylor second order (2) and Monte-Carlo simulations which are the reference method.

III. PROPAGATION OF MOMENTS METHOD ACCURACY

A. Benchmark : Electrical actuator model

As a primary test, the Propagation of moments method and Monte-Carlo simulations have been compared on a design model benchmark. The model has been introduced in [6] where a solution is given. The electrical actuator model is characterized by 9 non-linear explicit equations, 21 design continuous parameters, 12 degrees of freedom. The details of this model will be developed in the full paper.

B. Modeling procedure

First, we select input parameters of electrical actuator model likely to vary such as geometrical parameters or material properties. For Monte-Carlo simulations we arbitrarily choose normal distribution for each input parameter. Values for parameter's Mean are extracted from [6]. We test the accuracy of these methods for various Standard Deviations.

C. Results

All output parameters Means and Standard Deviations are computed using both Monte-Carlo simulations and the Propagation of moments method. Table I presents Monte-Carlo method for several simulation numbers (10^3, 10^4, 10^5, 10^6) and for the output parameter V_u (useful part volume of the electrical device). We give in the first column the input parameters Standard Deviations σ_i chosen as a percentage of their Means. We take the Monte-Carlo with 10^6 simulations as our reference for μ and σ. We express the relative error for each method we studied.

Therefore, Table II shows that Propagation of moments method using Taylor second order gives results close to those obtained using 10^6 simulations based Monte-Carlo (error max 0.083%). It appears that analytical expressions for μ and σ (2) are highly accurate.

IV. CONCLUSIONS AND PERSPECTIVES

According to our study, Propagation of moments method is accurate enough for optimization purpose.

The Table II gives criteria to select the appropriate robust analysis method regarding the problem encountered.

Criterion	Monte-Carlo simulations	Propagation of moments method
Use of deterministic optimization	No	Yes
Use of stochastic optimization	Yes	Yes
Evaluation number	10^6	2
Need to compute derivatives	No	Yes

TABLE II

ADVANTAGES AND DRAWBACKS OF THE TWO STUDIED METHODS

In our case we will use deterministic optimization methods to get a robust design. For this purpose, exact derivatives have to be evaluated and model evaluation number must be as fewer as possible. All these criteria lead us to choose the Propagation moments method.

Various studies and discussions will be given in the extended paper, about the sensitivity of the Propagation of moments expressions when used in robust optimization problems.

V. REFERENCES

[1] P. Breitkopf, H. Naceur, and A. Rassineux, P. Villon, « Moving least squares response surface approximation : Formulation and metal forming applications », *Computers and Structures*, vol. 83, issue 17-18, pp. 1411-1428, 2005.

[2] M. Lemaire, "Fiabilité des structures" *Hermès- Lavoisier*, 2005.

[3] R. Edwards, A. Marvin, and S. Poter, "Uncertainty analyses in the finite-difference time-domain method", *IEEE Transactions on Electromagnetic Compatibility*, vol. 52, issue 1, pp. 155-163, 2010.

[4] S. Rahman and H. Xu, "A univariate dimension-reduction method for multi-dimensional integration in stochastic mechanics", *Probabilistic Engineering Mechanics*, vol. 19, issue 4, pp. 393-408, 2004.

[5] J. Lei, P. Lima-Filho, M. A. Styblinski, and C. Singh, "Propagation of variance using a new approximation in system design of integrated circuits", *NAECON 1998. Proceedings of the IEEE National In Aerospace and Electronics Conference*, pp. 242-246, 1998.

[6] A.D. Kone, B. Nogarede, and M. Lajoie Mazenc, "Le dimensionnement des actionneurs électriques : un problème de programmation non linéaire", *Journal de Physique. III*, vol. 3, issue 2, pp. 285-301, 1993.